The AB questions may be found at:
http://apcentral.collegeboard.com/apc/public/repository/ap2012_calculusab_frq.pdf

AB1/BC1)

a) Estimate W'(12) [an instantaneous rate of change] with an average rate of change.

So $W'(12) \approx \dfrac{W(15)-W(9)}{15-9} = \dfrac{67.9-61.8}{6}* = \dfrac{6.1}{6} = 1.0166$ degrees/minute. (and you could stop at the asterisk). (Probably +1)

At t=12 minutes, the temperature of water in the tub is rising at the rate of 1.0166 degrees F per minute. (+1)

b) $\int_0^{20} W'(t)dt = W(20)-W(0)$ (FTC) $= 71.0-55.0* = 16.0$ degrees F. (+1)

$\int_0^{20} W'(t)dt$ is the total rise in temperature of the water in the tub, measured in degrees Fahrenheit, from t=0 to t=20 minutes. (+1)

c)

$\dfrac{1}{20}\int_0^{20} W(t)dt \approx \dfrac{1}{20}\left[(4-0)(55.0)+(9-4)(57.1)+(15-9)(61.8)+(20-15)(67.9)\right]* = 60.79$ degrees F. (+1)

A left Riemann sum should **underestimate** the actual average temperature of the water (+1) in the tub because the temperature is increasing over the first 20 minutes. The temperature is increasing because the water is being heated. (+1)

d) The temperature of the water at t=25 minutes is given by

$71.0+\int_{20}^{25} W'(t)dt = 71.0+\int_{20}^{25} 0.4\sqrt{t}\cos(0.06t)dt = 73.043$ degrees F. (+2)

AB2)

a) Find intersection: $\ln(x) = 5-x \Rightarrow x = 3.6934414;\ y = 1.306558652$

integrate with respect to y

$y=\ln(x) \Rightarrow x = e^y;\ y = 5-x \Rightarrow x = 5-y$

$Area = \int_0^{1.306558652} (5-y)-\left(e^y\right)dy = 2.985804$

Integrate with respect to x

$Area = \int_1^{3.6934414} \ln(x)dx + \int_{3.6934414}^{5} 5-x\,dx = 2.985804$

(Probably: point of intersection +1; limits +1; integrand +1; answer +1)

b) $Volume = \int_1^{3.6934414} \left(\ln(x)\right)^2 dx + \int_{3.6934414}^{5} (5-x)^2 dx = *2.7840177$

(limits +1; integrands +2; answer was not required)

c) points of intersection:
$\ln(x) = k \Rightarrow x = e^k$; $5 - x = k \Rightarrow x = 5 - k$
Integrate with respect to y
$\int_0^k (5-y)-(e^y)dy = \int_k^{1.306558652} (5-y)-(e^y)dy$ =2.985804/2
Integrate with respect to x
$\int_1^{e^k} \ln x dx + k((5-k)-(e^k)) + \int_{5-k}^{5} 5-x\,dx = \int_{e^k}^{3.6934414} \ln(x)\,dx + \int_{3.6934414}^{5} 5-x\,dx$ =2.985804/2
(+2)

AB3/BC3)
a) You know $g(1) = 0$.

You also know $g(2) = \int_1^2 f(t)dt$, which is also the net signed area between f(x) and the x-axis. The area can be calculated from the area of the triangle = $\frac{1}{2}(1)\left(\frac{1}{2}\right)$. The definite integral, however, is negative. So $g(2) = -\frac{1}{4}$.

$g(-2) = \int_1^{-2} f(t)dt = -\int_{-2}^{1} f(t)dt$. But this is the net signed area ….
So, $g(-2) = -\left[\frac{1}{2}(1)(3) - \frac{1}{2}\pi(1)^2\right] = \frac{\pi-3}{2}$.
(+1 for each value)

b) $g'(x) = f(x)$ by the FTC.
So $g'(-3) = 2$ from the graph.
$g''(x)$ is just the slope of the line segment in *f*, so $g''(-3) = 1$.
(+1 for each answer)

c) $g(x)$ has a horizontal tangent line when the value of the derivative is 0. I.E. $x = -1, +1$.

$g(x)$ will have a relative maximum if the derivative changes from positive to negative at the *x*-value, and will have a relative minimum if the derivative changes from negative to positive. But the derivative of *g* is just the function $f(x)$. The values of *f* change from positive to negative at *x* = -1, so at that *x*-value, *g* has a relative maximum. The values of *f* go from negative to 0 to negative at *x*= 1, so there is not an extremum at *x* = 1.
(+1 for max at x=-1; +1 for nothing at x=+1; +1 for reason)

d) $g(x)$ will have a point of inflection when the concavity changes. This will happen when

* $g''(x)$ is zero or undefined. The only time this happen is when $g'(x)$ has a horizontal tangent or is not differentiable (in this case, because of a cusp or corner). So we get these candidates for a point of inflection: x = -2, -1, 0, 1.

* those candidates identified in the first (*) either have $g''(x)$ changing sign OR $g'(x)$ changing from increasing to decreasing or vice-versa.

So $g(x)$ has a point of inflection at $g(-2), g(0), g(1)$. There is no point of inflection at $g(-1)$.
(+1 identifying the POIs; +1 explanation including rejecting x=-1).

This is a rich question – I could easily assign 11 or 12 points to this ….

AB4)

a) $f'(x) = \dfrac{-x}{\sqrt{25-x^2}}$ (+1 power rule; +1 chain rule)

b) $f(-3) = 4;\ f'(-3) = \dfrac{-(-3)}{4} = \dfrac{3}{4}$

So the equation of the tangent line is: $y - 4 = \dfrac{3}{4}\left(x - \left(-3\right)\right)$ (+1)

c) $\begin{array}{l} \lim\limits_{x\to -3^-} g(x) = \lim\limits_{x\to -3^-} \sqrt{25-x^2} = 4 \\ \lim\limits_{x\to -3^+} g(x) = \lim\limits_{x\to -3^+} x+7 = 4 \end{array}$ … so the two sided limit exists and equals $g(-3) = 4$ and therefore $g(x)$ is continuous at x = -3.
(+1 each limit; +1 conclusion; +1 citing or employing definition of continuity at a point)

d) $\int_0^5 x\sqrt{25-x^2}\,dx = \left\{u = 25 - x^2\right\} = -\dfrac{1}{2}\int_{25}^{0} u^{1/2}\,du = +\dfrac{1}{3}u^{3/2}\Big|_0^{25} = \dfrac{125}{3}$ (+2)

This makes one think that a horrible problem was discovered with the original #4 right before the printing of the exam, and they threw this in at the last minute. They didn't do teachers any favor by releasing this one …. (Or they are re-calibrating the difficulty of the exam).

AB5/BC5)

a) $\left.\dfrac{dB}{dt}\right|_{B=40} = \dfrac{1}{5}\left(100-40\right) = 12;\quad \left.\dfrac{dB}{dt}\right|_{B=70} = \dfrac{1}{5}\left(100-70\right) = 6$

Since $\dfrac{dB}{dt}$ represents the rate at which the bird is gaining weight, the bird is gaining weight faster at B(t)=40 because the rate of change is higher 12g/day vs. 6g/day).
(+1 answer; +1 reasoning)

b) $\dfrac{d^2B}{dt^2} = \dfrac{d}{dt}\left(20 - \dfrac{B}{5}\right) = -\dfrac{\frac{dB}{dt}}{5} = -\dfrac{100-B}{25} = \dfrac{B}{25} - 4$. Therefore, B is always concave DOWN (for B<100) and can not match the given graph, since the given graph changes concavity from UP to DOWN at approximately B(t) = 60 g.
(+1 second derivative; +1 reasoning)

c)
$$\frac{dB}{dt} = \frac{1}{5}(100 - B)$$
$$\frac{dB}{100 - B} = \frac{dt}{5}$$
$$-\ln(100 - B) = \frac{t}{5} + C$$
$$100 - B = e^{-\frac{t}{5} + C}$$
$$100 - 20 = e^{-\frac{0}{5} + C} = e^C; \quad \therefore C = \ln(80)$$
$$100 - B = 80e^{-\frac{t}{5}}; \quad \therefore B = 100 - 80e^{-t/5}$$
(+1 each for: separation of variables; each antiderivative; calculating C; final equation)

AB6)

a) $\cos\left(\dfrac{\pi}{6}t\right)$ is 0 when:

$\dfrac{\pi}{6}t = \dfrac{\pi}{2} \Rightarrow t = 3$

$\dfrac{\pi}{6}t = \dfrac{3\pi}{2} \Rightarrow t = 9$ and so on.

So the particle is moving left (when velocity is negative) on the interval (3,9).
(+2)

b) total distance = $\displaystyle\int_0^6 \left|\cos\left(\frac{\pi}{6}t\right)\right| dt$

(+2)

c) $a(t) = v'(t) = -\dfrac{\pi}{6}\sin\left(\dfrac{\pi}{6}t\right)$.

$$a(4) = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}(4)\right) = -\frac{\pi}{6}\sin\left(\frac{2\pi}{3}\right) < 0$$

Since acceleration is negative, and velocity is negative (see part a), the particle is speeding up.

(+1 each: a(t), conclusion, explanation)

d) position (at t=4) = initial position + total change in position =

$$-2+\int_0^4 \cos\left(\frac{\pi}{6}t\right)dt = -2+\frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\Big|_0^4 = -2+\frac{6}{\pi}\left(\sin\left(\frac{2\pi}{3}\right)-\sin(0)\right) = {}^*-2+\frac{3\sqrt{3}}{2}$$

(+1 initial position, +1 displacement)

The BC questions may be found at:
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BC2

a) The horizontal movement of the particle is movement in the y direction … so we want $\frac{dy}{dt}$. $\frac{dy}{dt}\Big|_{t=2} = \sin^2 2 > 0$. Therefore the particle is moving to the right.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin^2 t}{\left(\frac{\sqrt{t+2}}{e^t}\right)}.\ \text{When t=2, } \frac{dy}{dx} = \frac{\sin^2 2}{\left(\frac{\sqrt{2+2}}{e^2}\right)} = \frac{e^2\sin^2 2}{2} = {}^*3.054716.$$

(+1 move to right; +1 explanation; +1 slope)

b) $x(4) = x(2)+\int_2^4 \frac{\sqrt{t+2}}{e^t}dt = 1+.252954 = 1.252954$

(+1 initial position, +1 change)

c) speed = magnitude of velocity vector = $\sqrt{\left(\frac{\sqrt{4+2}}{e^4}\right)^2+\left(\sin^2(4)\right)^2} = {}^*.5745$

$$a(4) = \left(\frac{d^2x}{dt^2}\Big|_{t=4}, \frac{d^2x}{dt^2}\Big|_{t=4}\right) = (-0.0411, 0.989358)$$

(+1 each)

d) distance traveled by particle is Arc Length …

which is $\int_2^4 \sqrt{\left(\frac{\sqrt{t+2}}{e^t}\right)^2+\left(\sin^2(t)\right)^2}\,dt = 0.650983$

(+1 integral, +1 answer)

BC4

a) $y-15 = 8(x-1)$. If x = 1.4 then y on the tangent line is $8(1.4-1)+15 = 18.2$

(+1 each)

b) $\int_1^{1.4} f'(x)dx \approx (1.2-1)(f(1.1))+(1.4-1.2)(f(1.3)) = .2(10)+.2(13) = 4.6$

$f(1.4) = f(1) + \int_1^{1.4} f'(x)dx = 15 + 4.6 = 19.6$
(+1 for each)

c) $f(1) = 15$
$f(1.2) \approx f(1) + f'(1)(0.2) = 15 + 8(.2) = 16.6$
$f(1.4) \approx f(1.2) + f'(1.2)(0.2) = 16.6 + 12(.2) = 19.0$
(+2)

d) $T_2(x) = 15 + 8(x-1) + \frac{20}{2!}(x-1)^2$
$T_2(1.4) = 15 + 8(1.4-1) + \frac{20}{2!}(1.4-1)^2 = *15 + 3.2 + 15(.16) = 20.6$
(+1 each)

BC6

a) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{2n+3}}{2n+5}\cdot\frac{2n+3}{x^{2n+1}}\right| = \left|x^2\right|.$ The ratio test says the series converges when the $\left|x^2\right| < 1 \Rightarrow -1 < x < 1$
limit is less than 1.
Checking endpoints gives us (in each case) a series comparable to the alternating harmonic series. So the actual interval of convergence is $-1 \le x \le 1$.
(+1 ratio test, +1 for each endpoint checked; +1 interval of convergence)

b) The absolute value of the error in the n^{th} partial sum of an alternating series is less than the absolute value of the next term (provided the limit of the terms is 0).

$|a_3| = \left(\frac{1}{2}\right)^5 / 7 = \frac{1}{32\cdot 7} = \frac{1}{224} < \frac{1}{200}$.
(+1 demonstration; +1 appeal to alternating series error test)

c) $g'(x) = M(x) = \frac{1}{3} - \frac{3x^2}{5} + \frac{5x^4}{7} - \ldots + \frac{(-1)^n(2n+1)(x)^{2n}}{2n+3} + \ldots$
(+1 first three terms; +2 for general term)

How did you do?